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ON STABILITY OF FREE-FREE BEAMS WITH AND WITHOUT DIRECTIONAL CONTROL

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December 1976

US ARMY ARMAMENT COMMAND

BENET WEAPONS LABORATORY
WATERVLIET ARSENAL
WATERVLIET, N.Y. 12189

TECHNICAL REPORT

AMCMS No. 611102.11.H4500

DA Project No. 1F161102AH45

Pron No. EJ-T-Y0019-01-EJ-M7

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REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER WVT-TR-76049	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (*and Subtitle*) On Stability of Free-Free Beams With and Without Directional Control		5. TYPE OF REPORT & PERIOD COVERED 6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) J. J. Wu		8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS Benet Weapons Laboratory Watervliet Arsenal, Watervliet, N.Y. 12189 SARWV-RT-TP		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS AMCMS No. 611102.11.H4500 DA Proj. No. 1F161102AH45 Pron No. EJ-T-Y0019-01-EJ-M7
11. CONTROLLING OFFICE NAME AND ADDRESS US Army Armament Command Rock Island, Illinois 61201		12. REPORT DATE December 1976 13. NUMBER OF PAGES 18
14. MONITORING AGENCY NAME & ADDRESS (*if different from Controlling Office*)		15. SECURITY CLASS. (*of this report*) UNCLASSIFIED 15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (*of this Report*)

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17. DISTRIBUTION STATEMENT (*of the abstract entered in Block 20, if different from Report*)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (*Continue on reverse side if necessary and identify by block number*)

Directional Control
Dynamic Stability
Eigenvalue
Finite Element
Flutter
Free-Free Beams
Vibration

20. ABSTRACT (*Continue on reverse side if necessary and identify by block number*)

This report presents some new findings and revised data concerning the stability of a uniform free-free beam subjected to a constant thrust with and without directional control. An analysis is performed first showing that, other than the first branch of zero eigenvalues which correspond to rigid body translation modes, there can be only a set of discrete values of the thrust which admit zero eigenvalues. Then, stability data, revised from previous results with a numerical error eliminated, are presented. Conclusions on the beam's stability are drawn from these new evidences.

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INTRODUCTION

Concerning the stability of a uniform free-free beam under constant thrust with and without directional control, the second lowest branch of eigenvalues turns out to be the critical one in deciding the structural stability as it has been reported by the writer in several previous papers [1,2,3]. This fact was not mentioned in earlier investigations [4,5,6]. However, due to a numerical error as pointed out by Sundararamaiah and Johns [7], some of the writer's conclusions also become invalid. This note attempts to clarify some aspects of this crucial branch of eigenvalues. First, an analysis is presented showing that, other than the first branch of zero eigenvalues (corresponding to the rigid body translation mode), there can be only a set of discrete values of the thrust which admit zero eigenvalues. Stability data are then presented which are revised from previous results with the numerical error eliminated. Finally conclusions concerning the second lowest branch of eigenvalues are drawn from these new evidences.

ON DISCRETE VALUES OF THE THRUST WHICH ADMIT ZERO EIGENVALUES

Referring to notations adapted in [1], the lateral stability of a uniform free-free beam under a constant thrust subjected to directional control can be described by the following boundary value problem:

$$\text{D.E.} \qquad u'''' + Q(xu')' + \lambda^2 u = 0, \qquad (1a)$$

B.C. $u''(0) = u'''(0) = 0$ (1b,1c)

$u''(1) = 0, \ u'''(1) - K_\theta Q u'(1) = 0$ (1d,1e)

As mentioned in [1], Q is the non-dimensionalized thrust, λ, the eigenvalue which dictates the stability behavior and K_θ, the directional control parameter. A positive K_θ denotes a rotation of the thrust in the same direction as the tangent at the tail-end and a negative K_θ, in opposition to the same tangent.

First we shall present here an analysis showing that, other than the rigid body translation mode for which the eigenvalue λ is zero for all values of Q, there can only be a set of discrete values of Q, designated by Q_1, i=0,1,2,....., at which λ take zero values and that Q_i's are independent of K_θ. For this purpose, set λ=0 in eqs (1) and one has

D.E. $u'''' + Q(xu')' = 0$ (2a)

B.C. $u''(0) = u'''(0) = 0$ (2b,2c)

$u''(1) = 0, \ u'''(1) - K_\theta Q u'(1) = 0$ (2d,2e)

Let

$v = u'$ (3)

Eqs (2) become

D.E. $v''' + Q(xv)' = 0$ (4a)

B.C. $v'(0) = v''(0) = 0$ (4b,4c)

$v'(1) = 0, \ v''(1) - K_\theta Q v(1) = 0$ (4d,4e)

From eq (4a) one has

$$v'' + Qxv = \text{constant} = c$$

The constant c must vanish due to eq (4c). In order to satisfy eq. (4e), one can simply replace eqs (4) by the following system:

$$\text{D.E.} \qquad v'' + Q[x - \alpha_\theta \varepsilon\, \delta(x - 1)]v = 0 \tag{5a}$$

$$\text{B.C.} \qquad v'\,(0) = v'\,(1) = 0 \tag{5b,5c}$$

where

$$\alpha_\theta = 1 + K_\theta \tag{6}$$

$\delta(x)$ is the Dirac delta function and ε is an infinitesimal number defined by

$$\varepsilon\delta(0) = 1 \tag{7}$$

It is an easy matter to verify the equivalence between eqs (4) and (5). In particular, substituting x = 1 in eq. (5a), one has

$$v''\,(1) + Q(1 - \alpha_\theta)\, v'\,(1) = 0$$

which is eq (4e) by noting eq (6).

To solve eqs (5), the method of finite element-unconstrained variations [2] is again used here. It is hardly necessary to describe the details since they follow so closely as in [2]. Only the unconstrained variational statement is given here. Through integration-by-parts, it is easily shown that the one associated with eqs (5) is

$$\delta J = 0 \tag{8a}$$

where

$$J(v,v^*) = \int_0^1 (-v'v^{*\prime} + Qvv^*)dx + \varepsilon\alpha_\theta Qv(1)v^*(1) \tag{8b}$$

and v^* is the adjoint variable. Note that v and v^* are symmetric in eq (8b) indicating that eqs (5) is now a self-adjoint problem. The significance that ε is infinitesimally small (eq (7)) is now apparent. As long as α_θ is a finite, the solutions for Q in eqs (8) are independent of α_θ (or, $K_\theta = \alpha_\theta - 1$). The eigenvalue problem of eqs (8) also indicates that, other than the trivial solution v=u'=0 (which corresponds to $\lambda=0$ for all values of Q), the discrete eigenvalues Q_i, i=0,1,2,....., are the only solutions. The first eight of Q_i's obtained are listed in Table 1.

TABLE 1. THE FIRST EIGHT LOWEST VALUES OF THE THRUST WHICH ADMIT ZERO EIGENVALUES (RIGID BODY TRANSLATION MODE EXCLUDED)

i	0	1	2	3	4	5	6	7
Q_i/π^2	0.0000	2.5677	9.7218	21.348	37.486	58.156	83.333	113.30

It might be worthwhile to mention a different way to obtain the Q_i's. Since the Q_i's are independent of $K_\theta = -1$ ($\alpha_\theta = 0$) in eqs (5), one then has* [8]

D.E. $$v'' + Qxv = 0 \tag{9a}$$

B.C. $$v'(0) = v'(1) = 0 \tag{9b,9c}$$

Eqs (9) is reducible to Bessel's equation (see, for example, [9]) and it can be readily shown that solutions to eqs (9) are those of

$$J_{2/3}\left(\frac{2}{3} Q^{1/2}\right) = 0 \tag{10}$$

---

*This is exactly the equations solved by Silverberg [8], although his interpretation of the result was incorrect as it was pointed out earlier by the writer [2].

where J in eq (10) is the Bessel's function of the first kind. The first non-zero solution of eq (10) turns out to be $2.5992\pi^2$ which agrees quite well with Q_1 given in Table 1.

STABILITY DATA REVISED

In this section, the stability data revised from previous results, some of which are misleading due to a numerical error, are presented. Figure 1 is for the case without control. The deformed parabola corresponds to the third and fourth lowest branch of eigenvalues (for the range of thrust shown). These two branches now coalesce at $Q = 11.126\pi^2$ with $\lambda = 23.10$ which agree with the results given in [7]. The first branch is zero for all values of Q and corresponds to the rigid body translation mode. The second branch is a troublesome one. From the present numerical results, for Q ranging from zero to $100\pi^2$, say, the eigenvalues of this branch are too small* to be distinguishable from the first one in which the eigenvalues are supposed to be true zeroes. On the other hand, the analysis of the previous section has shown that there is no zero eigenvalue solution other than those of Q_i. Thus we conclude that this second branch for $K_\theta = 0$ is an "almost zero" branch.

Figures 2 through 5 show the first four eigenvalue branches (including the zero branch) for $K_\theta \neq 0$. In the order of the figure

*For larger values of Q ($Q = 1000\pi^2$, for example), this branch appears to be distinguishable from zero again.

numbers 2,3,1,4 and 5, it is revealing to observe the changing trend of these curves as K_θ varies from -1.0 to -0.05, to 0, to 0.05 and to 1.0. The changing pattern of the "second" branch is of particular interest. It indicates that, for small $|K_\theta|$ (additional data include cases in which $|K_\theta|$ is as small as 0.0001), the second branch has a divergence region in $0<Q<Q_1$ so long as K_θ is negative and it becomes a stable region so long as K_θ is positive. Hence the case $K_\theta = 0$ appears to be bordering between stability and divergence instability. In case of divergence instability, the eigenvalue λ indicates the rate at which the structural disturbance will grow. Since the second branch λ is very small for $K_\theta = 0$, it may be considered a stable mode in the practical sense until the flutter load $Q = 11.126\pi^2$ is reached. However, unless $Q = 0$, this small λ does not represent a rigid body rotation mode and thus must be a blending mode.

Finally, Figure 6 shows the three lowest branches of eigenvalues for various values of K_θ and for Q between 0 and $4\pi^2$. It can be considered a revision of Figure 5 in [1].

CONCLUSIONS

Concerning the stability problem of a uniform free-free beam with and without directional control, we now may summarize the following:

1. Barring the rigid body translation mode, λ can be zero only at a discrete set of Q's (designated by Q_i, i=0,1,2,....., the first eight of them are given in Table 1). These Q_i's are independent of K_θ.

2. As long as K_θ is positive, there is a region of oscillatory

stability between Q_0 and Q_1.

3. As long as K_θ is negative, there is a region of divergence instability between Q_0 and Q_1.

4. For $K_\theta = 0$, the second branch of eigenvalues represents a bordering state of stability and instability. This branch may be considered stable in the practical sense due to the smallness of the λ's. It is a bending mode nevertheless.

It should be pointed out that Items 2 and 3 above are concluded from numerical results only and have not been substantiated by analytical proofs.

In retrospect, the writer was not justified to have stated in his previous papers that the methods used by Feodos'ev [5] and by Matsumoto and Mote [6] had led to incorrect results. He could only state that they did not discuss the important second branch. However, the Galerkin's procedure used by Beal [4], which results in a second zero branch for all values of Q, is still inconsistent with the analysis presented here.

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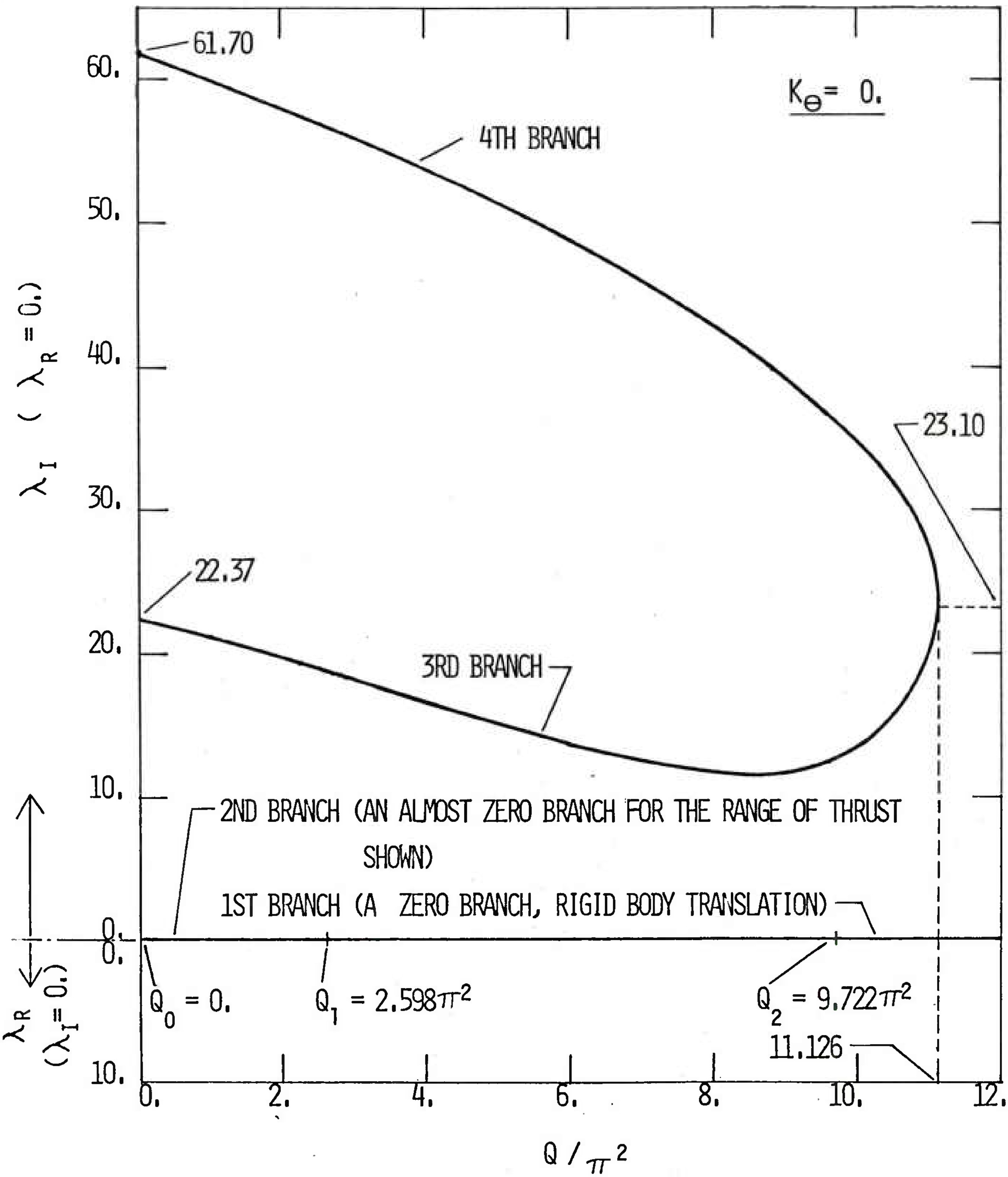

Figure 1. Four lowest branches of eigenvalues, K_θ = 0.

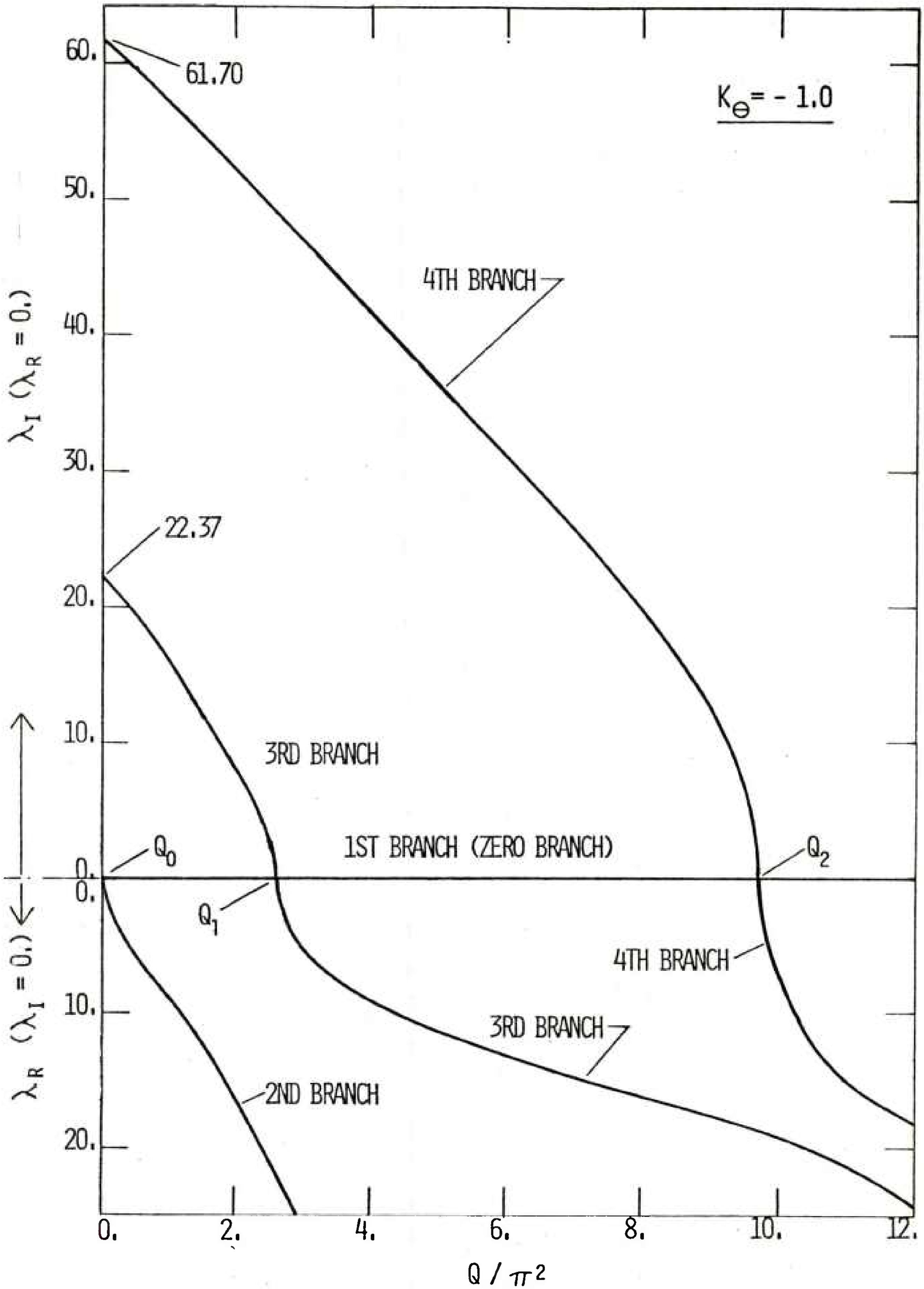

Figure 2. Four lowest branches of eigenvalues, $K_\theta = -1.0$.

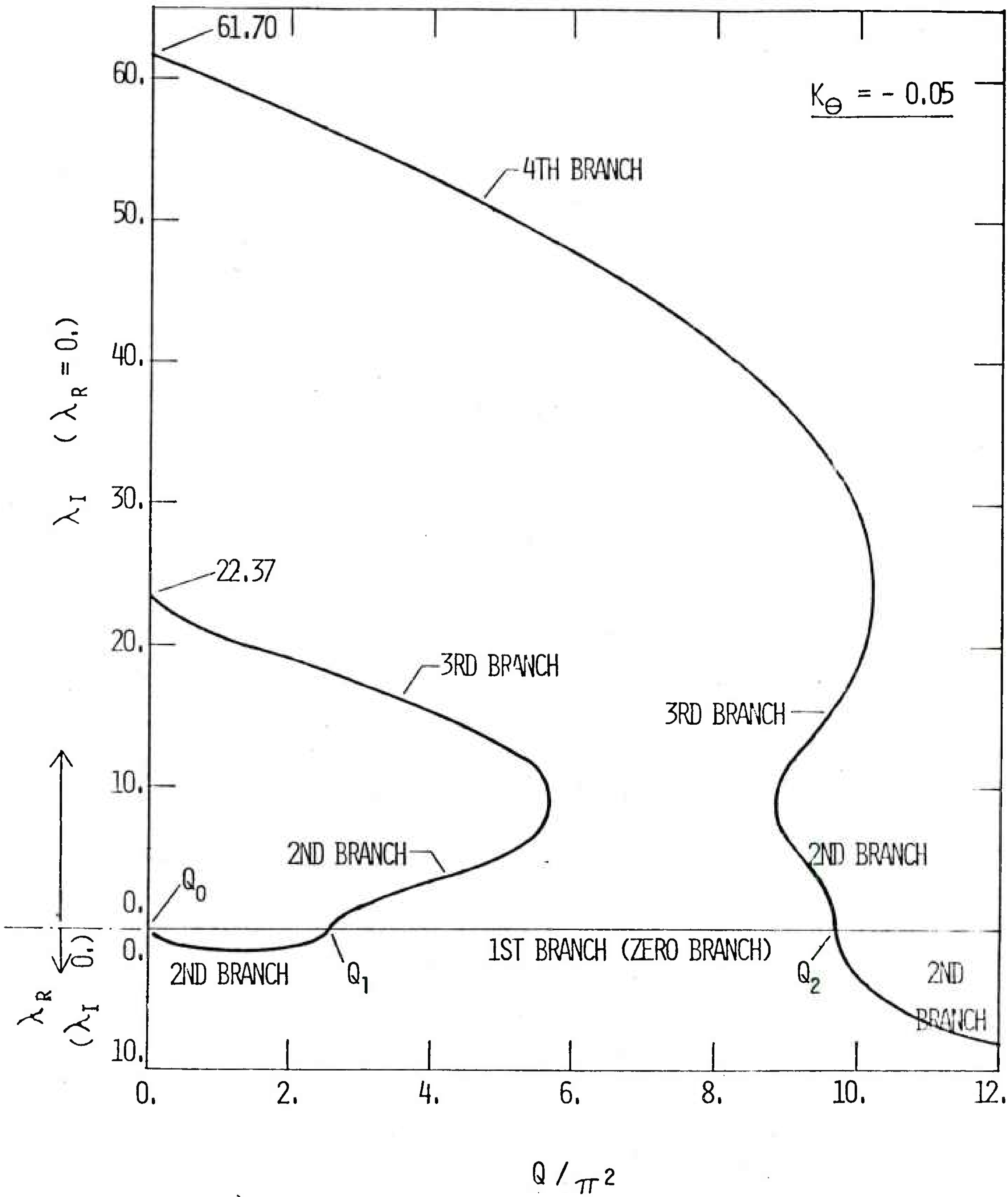

Figure 3. Four lowest branches of eigenvalues, K_θ = -0.05.

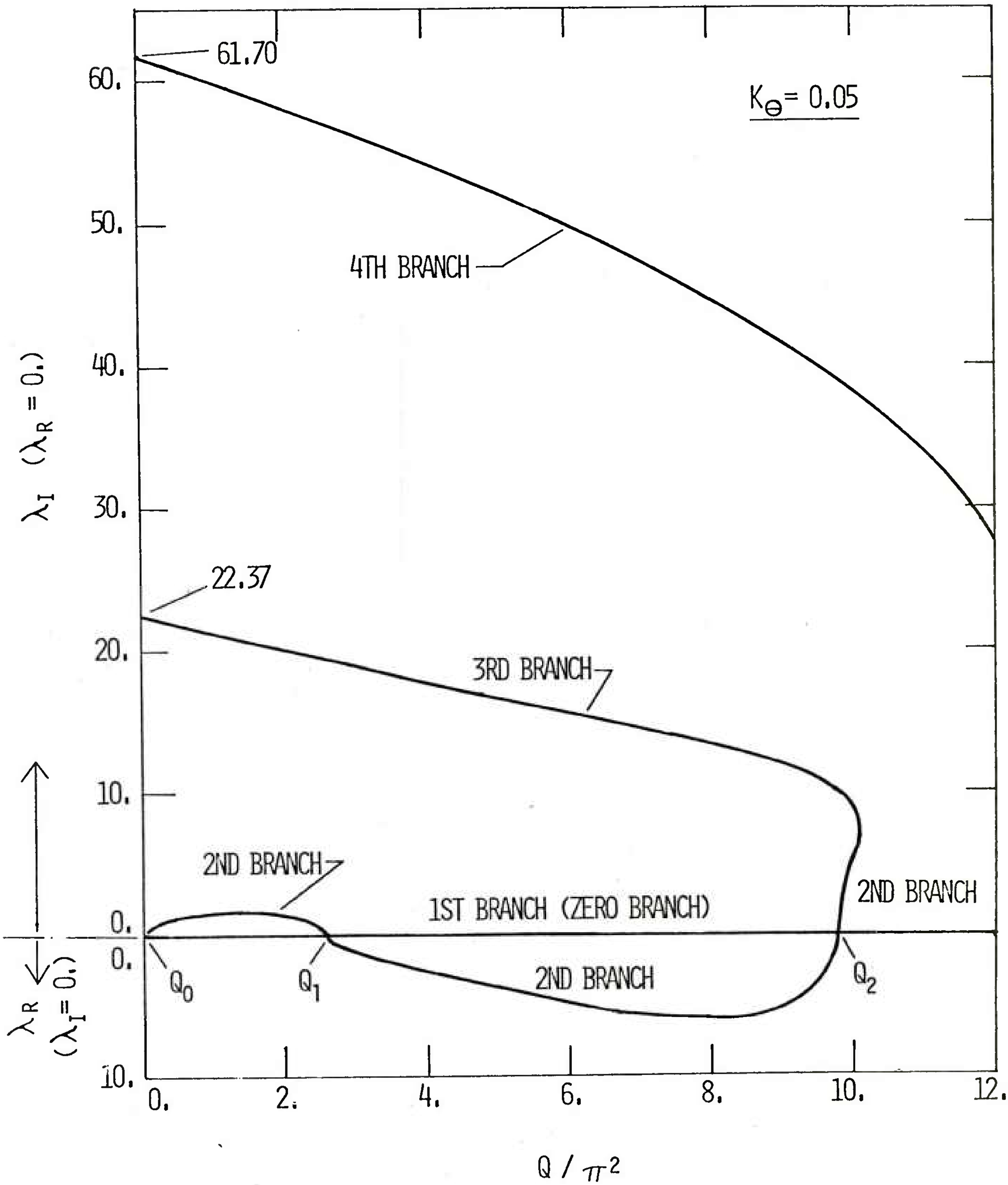

Figure 4. Four lowest branches of eigenvalues, K_θ = 0.05.

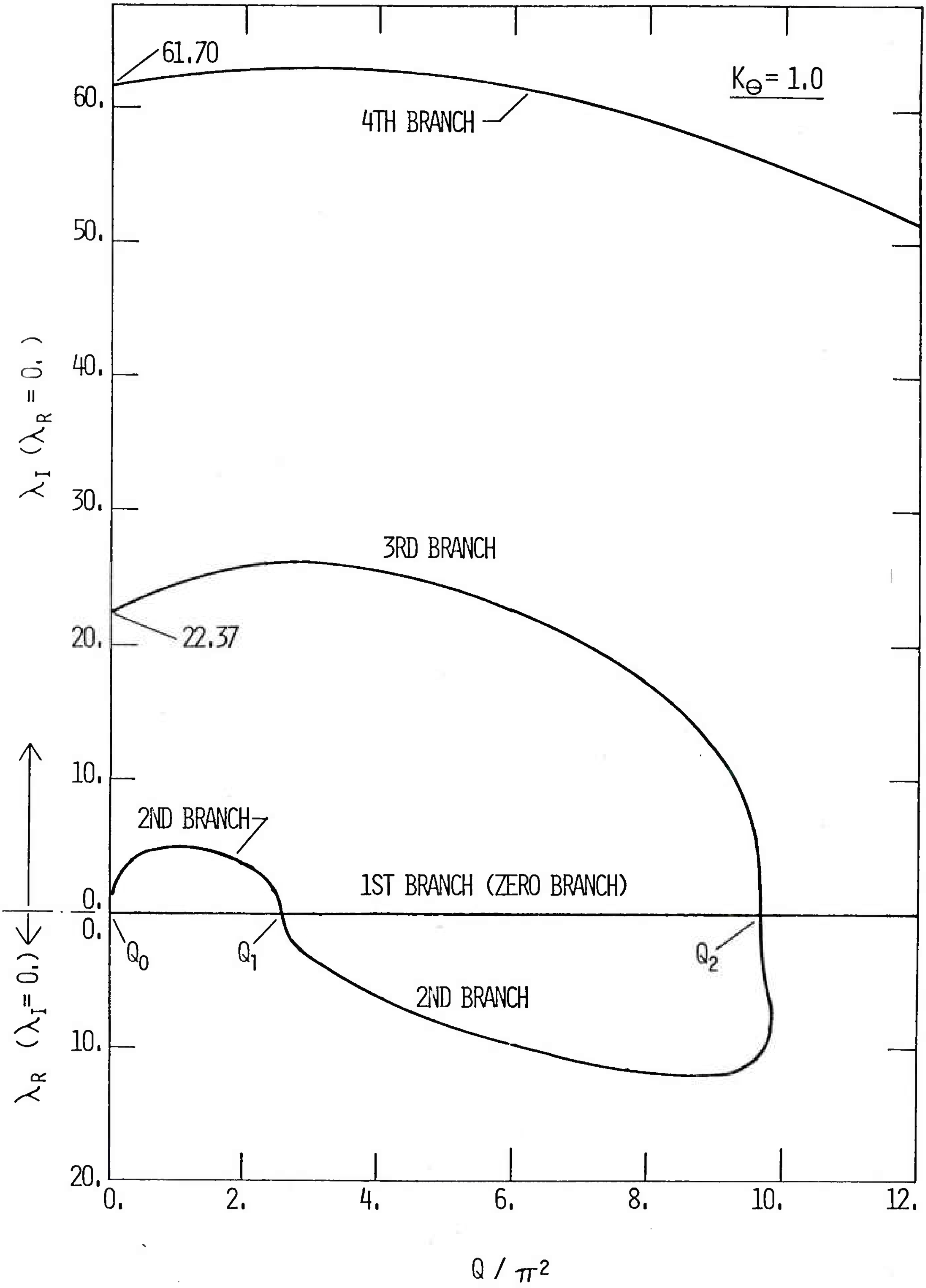

Figure 5. Four lowest branches of eigenvalues, K_θ = 1.0.

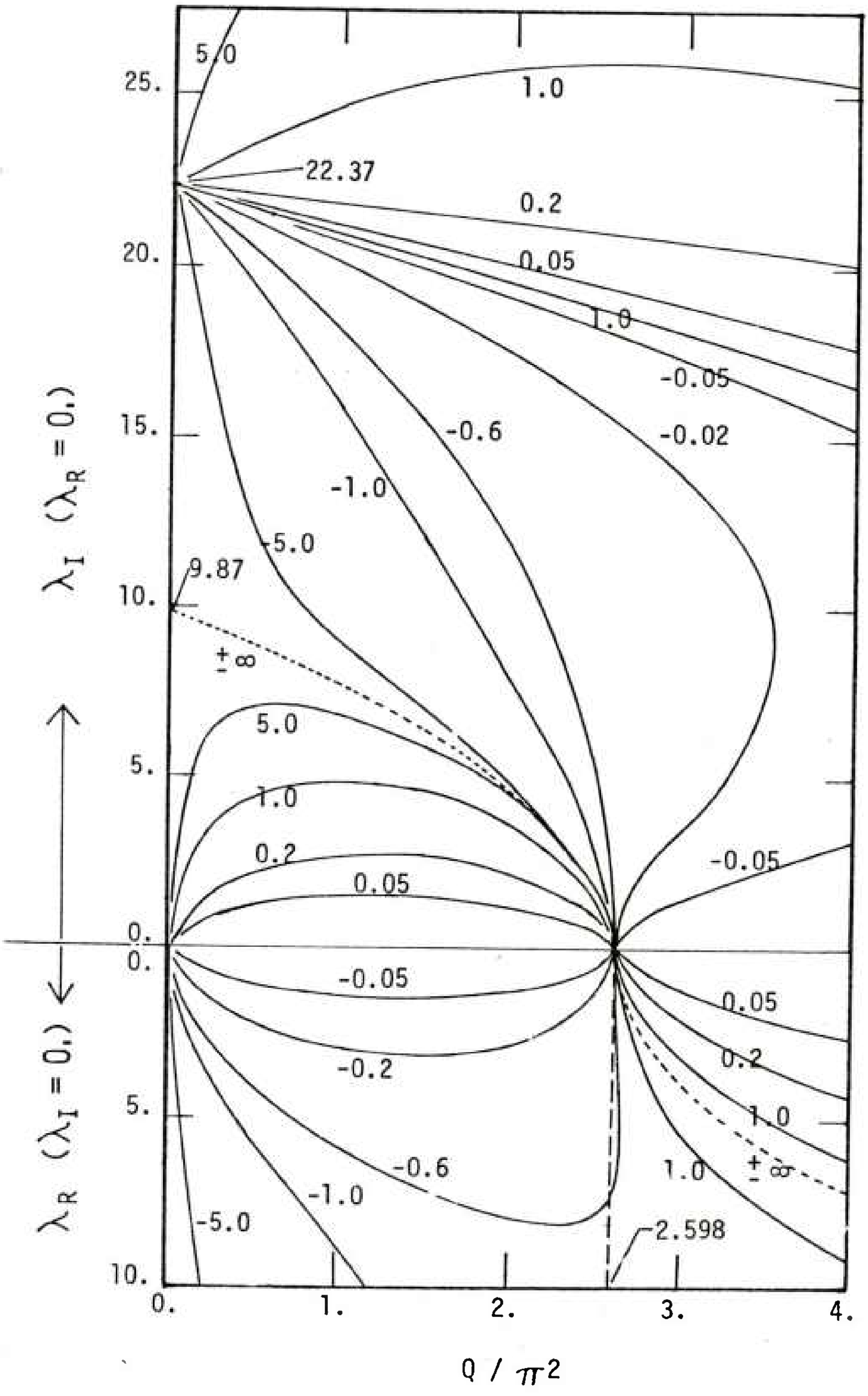

Figure 6. Three lowest branches of eigenvalues for various values of K_θ.

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